

# STATIC QUARK CORRELATORS IN SU(2) GAUGE THEORY AT FINITE TEMPERATURE

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April 25, 2008

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## Introduction

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# Introduction

Correlation function of static mesons is relevant for

- ▶ melting of the hadronic states in the deconfined phase
- ▶ color screening in the deconfined phase
- ▶ quarkonium dissociation

Previous studies almost exclusively use the Coulomb gauge.

# Static quark correlators

Static meson operators:

$$O(x, y; t) = \bar{\psi}(x, t) U(x, y; t) \psi(y, t), \quad (1)$$

$$O^a(x, y; t) = \bar{\psi}(x, t) U(x, x_0; t) T^a U(x_0, y; t) \psi(y, t). \quad (2)$$

Consider correlators of these operators at  $t = 1/T$  (after integration over the static fields):

$$G_1(r, T) = \frac{1}{2} \langle \text{Tr} L^\dagger(x) U(x, y; 0) L(y) U^\dagger(x, y, 1/T) \rangle, \quad (3)$$

$$G_3(r, T) = \frac{1}{3} \langle \text{Tr} L^\dagger(x) \text{Tr} L(y) \rangle \\ - \frac{1}{6} \langle \text{Tr} L^\dagger(x) U(x, y; 0) L(y) U^\dagger(x, y, 1/T) \rangle \quad (4)$$

# Static quark correlators

These correlators:

- ▶ depend on the choice of the spatial transporters  $U(x, y; t)$ ,
- ▶ in the special gauge, where  $U(x, y, z; t) = 1$  give standard definition of the singlet and triplet free energies

$$\exp(-F_1(r, T)/T + C) = \frac{1}{2} \langle \text{Tr} L^\dagger(x) L(y) \rangle, \quad (5)$$

$$\begin{aligned} \exp(-F_3(r, T)/T + C) &= \\ \frac{1}{3} \langle \text{Tr} L^\dagger(x) \text{Tr} L(y) \rangle - \frac{1}{6} \langle \text{Tr} L^\dagger(x) L(y) \rangle, \quad (6) \\ r &= |x - y|. \quad (7) \end{aligned}$$

# Static quark-antiquark free energies

The physical free energy of a static quark anti-quark pair is given by the thermal average of the singlet and triplet free energy

$$\begin{aligned} \exp(-F_a(r, T)/T) &= \\ \frac{1}{4} \exp(-F_1(r, T)/T) + \frac{3}{4} \exp(-F_3(r, T)/T) \\ &= \frac{1}{4} \langle \text{Tr} L(r) \text{Tr} L(0) \rangle. \end{aligned} \quad (8)$$

At high temperature in the leading order HTL approximation the singlet and triplet free energies are

$$F_1(r, T) = -\frac{3}{4} \frac{\alpha_s}{r} \exp(-m_D r) - \frac{3}{4} \alpha_s m_D \quad (9)$$

$$F_3(r, T) = +\frac{1}{4} \frac{\alpha_s}{r} \exp(-m_D r) - \frac{3}{4} \alpha_s m_D \quad (10)$$

# Static quark-antiquark free energies

Using the transfer matrix one can show that

$$G_1(r, T) = \sum_{n=1}^{\infty} c_n^1(r) e^{-E_n(r, T)/T}, \quad (11)$$

$$G(r, T) = \langle \text{Tr} L(r) \text{Tr} L(0) \rangle = \sum_{n=1}^{\infty} e^{-E_n(r, T)/T}, \quad (12)$$

where  $E_n$  are the energy levels of static quark and anti-quark pair. The coefficients  $c_n^1(r)$  depend on the choice of the transporters  $U$ .

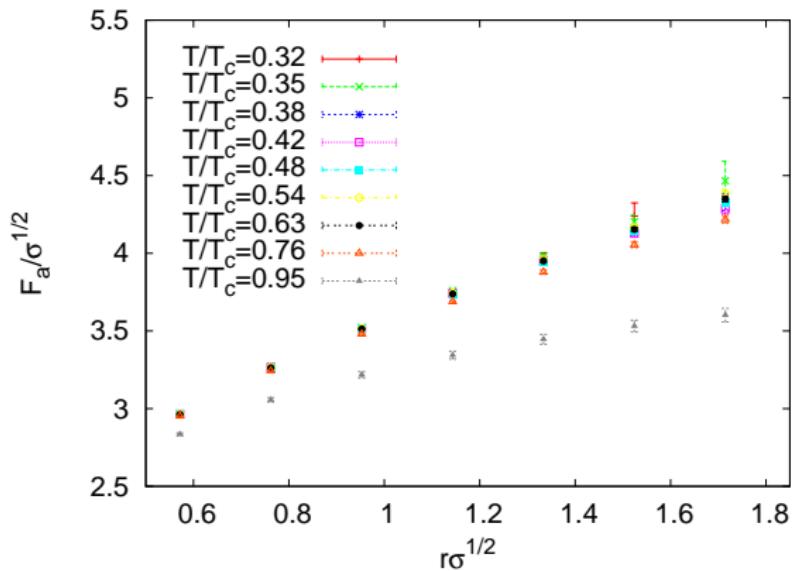
If  $c_1^1 = 1$  the dominant contribution to  $G_3$  would be the 1st excited state  $E_2$ , thus justifying the name singlet and triplet free energy.

In perturbation theory  $c_1^1 = 1$  up to  $\mathcal{O}(g^6)$  corrections<sup>1</sup> and therefore at short distances,  $r \ll 1/T$  the color singlet and color averaged free energy are related  $F_a(r, T) = F_1(r, T) + T \ln 4$ .

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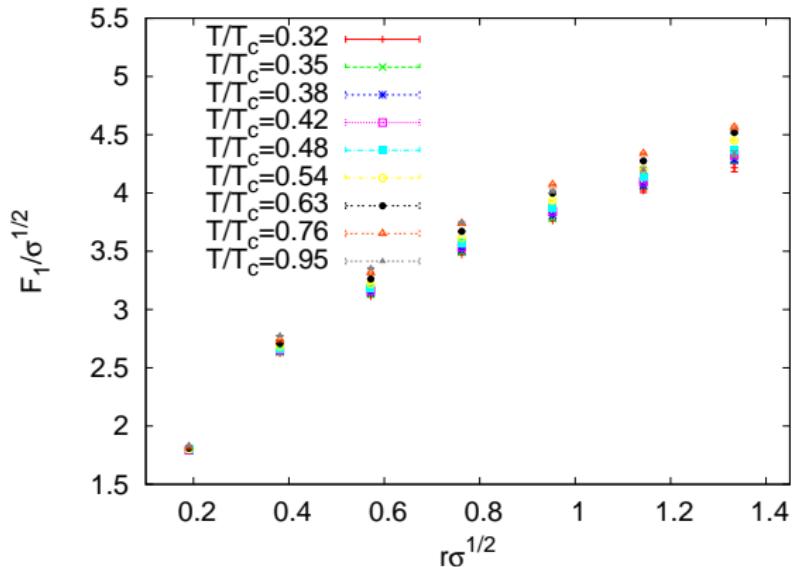
<sup>1</sup>N. Brambilla, A. Pineda, J. Soto, A. Vairo, NPB 566, 275 (2000)

# Color averaged free energy at $\beta = 2.5$



Almost no  $T$ -dependence up to  $T/T_c = 0.76$ .

# Color singlet free energy at $\beta = 2.5$



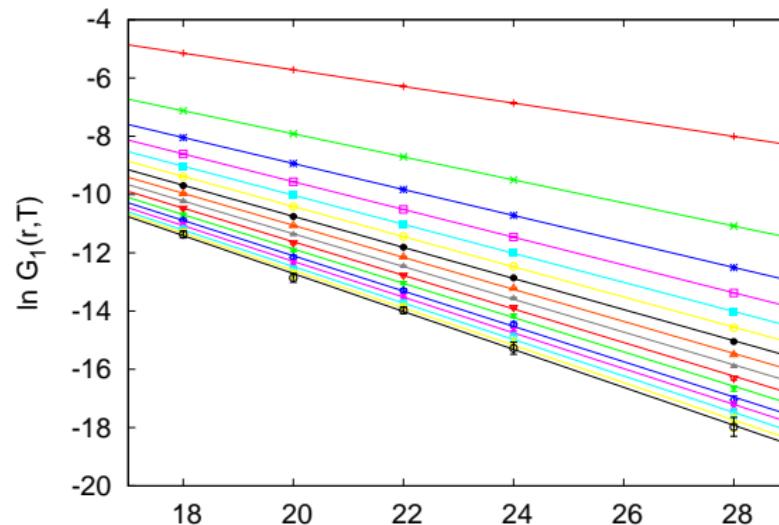
Almost no  $T$ -dependence up to  $T/T_c = 0.95$ .

# Extracting $c_1^1$ and $c_1^a$

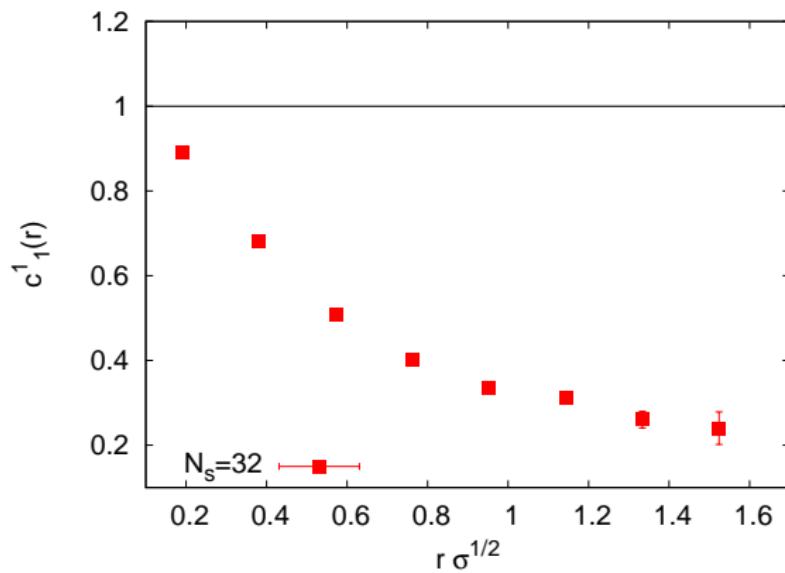
Truncate and fit at fixed  $r$ :

$$G_1(r, T) = \sum_{n=1}^{\infty} c_n^1(r) e^{-E_n(r, T)/T}$$

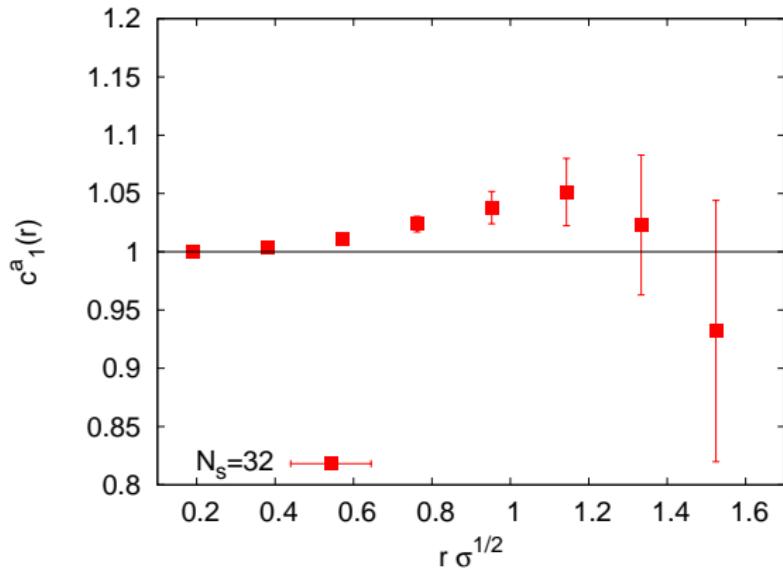
$$G_1(r, T) = c_1^1(r) e^{-a(r) N_\tau}$$



# Coefficient $c_1^1(r)$ at $\beta = 2.5$



# Coefficient $c_1^a$ at $\beta = 2.5$



# Problems with extracting triplet

If  $c_1^1 \neq 1$  then <sup>2</sup>

$$e^{-\tilde{F}_a(r)/T} = G(r, t) = c_1^a e^{-E_1(r)/T},$$

$$e^{-\tilde{F}_1(r)/T} = G_1(r, T) = c_1^1(r) e^{-E_1(r)/T},$$

$$e^{-\tilde{F}_3(r)/T} = c_1^3(r) e^{-E_1(r)/T}$$

and

$$\tilde{F}_a(r, T) = E_1(r, T) - T \ln c_1^a,$$

$$\tilde{F}_1(r, T) = E_1(r, T) - T \ln c_1^1(r),$$

$$\tilde{F}_3(r, T) = E_1(r, T) - T \ln c_1^3(r),$$

$$c_1^a \sim 1, \quad c_1^3(r) \sim 1 - c_1^1(r).$$

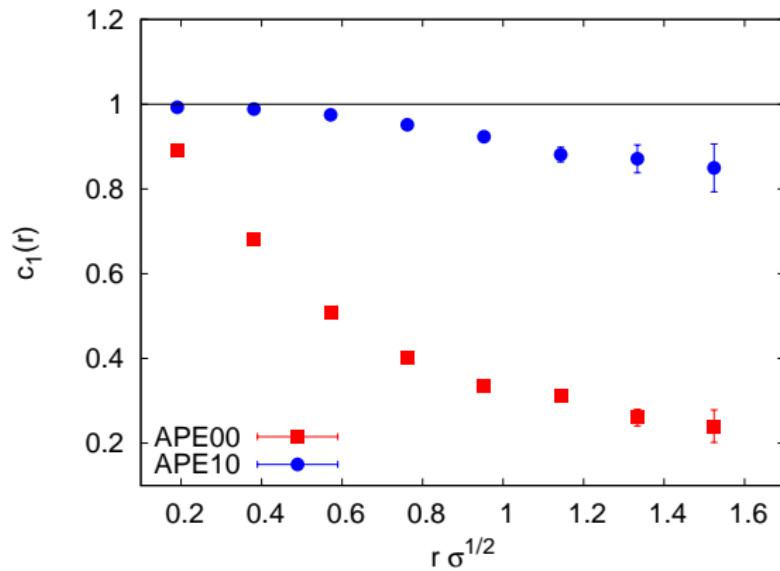
$\tilde{F}_3(r, T)$  has a contribution from the singlet!

<sup>2</sup>O. Jahn, O. Philipsen, PRD 70, 074504 (2004)

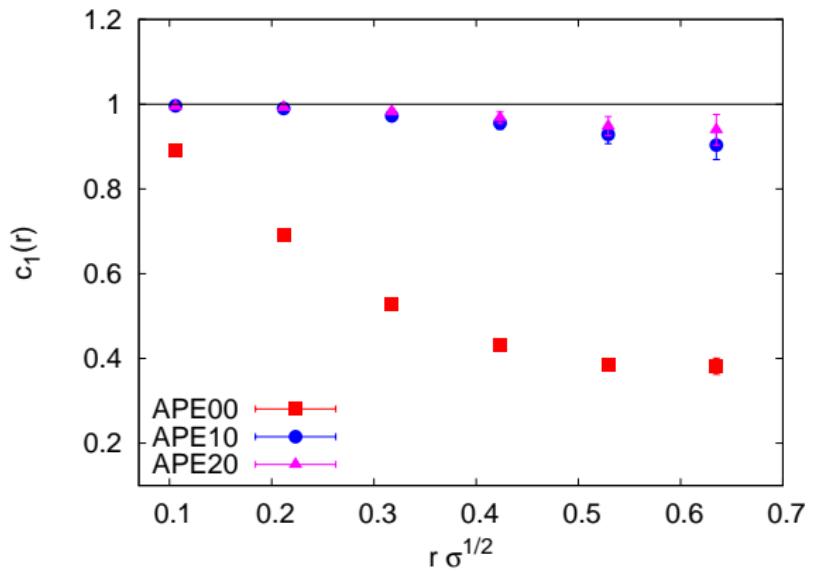
# Summary of the simulation

- ▶ 4D SU(2) gauge theory with Wilson action
- ▶ Updating with Lüscher-Weiss multilevel algorithm
- ▶ Lattices  $N_s^3 \times N_\tau$ ,  $N_s = 16, 24, 32$
- ▶ Iteratively use APE smearing procedure to construct the transporters  $U(x, y, z; t)$

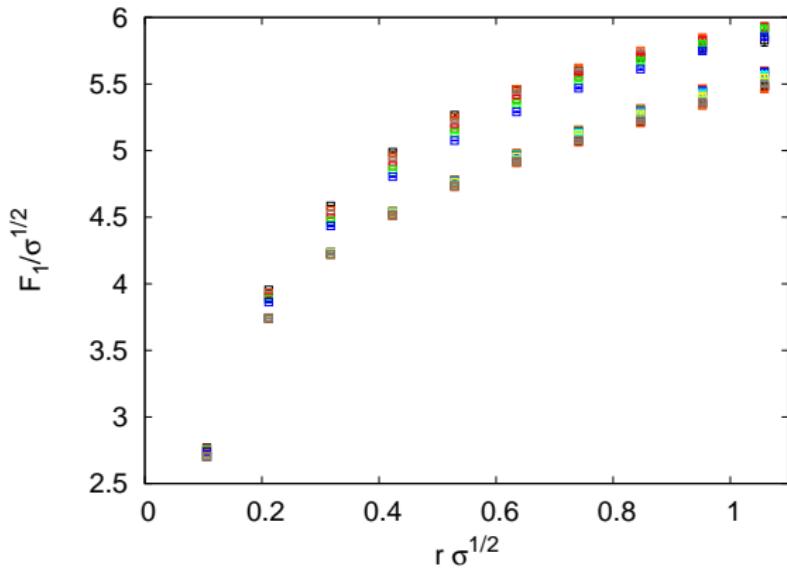
# Coefficient $c_1^1$ at $\beta = 2.5$



# Coefficient $c_1^1$ at $\beta = 2.7$

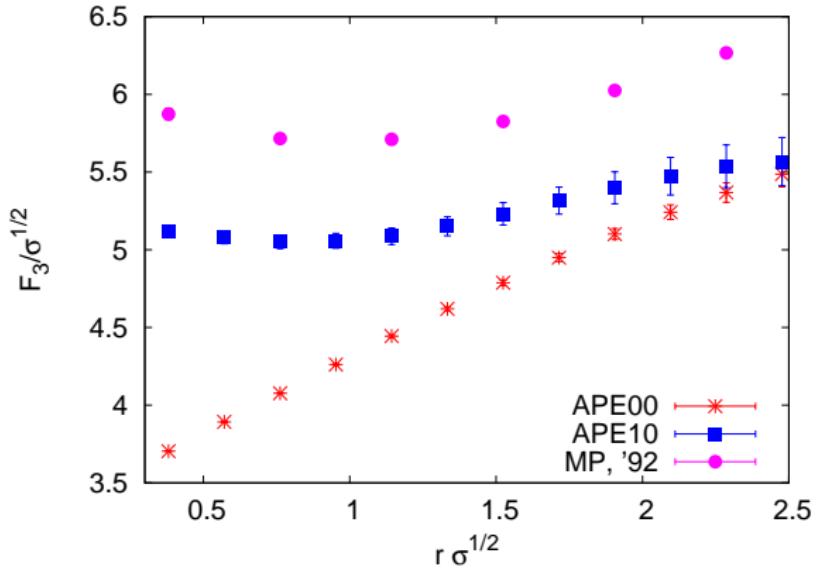


# Singlet free energy



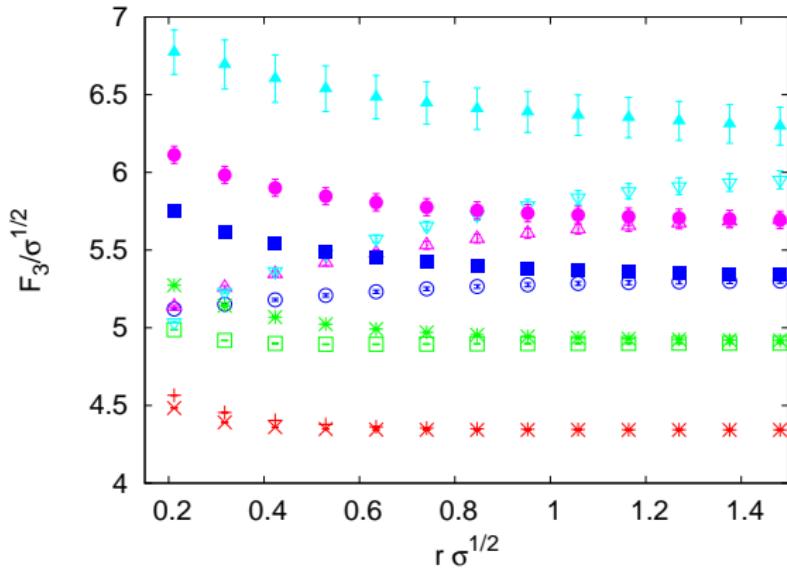
$T$ -dependence is greatly reduced with APE10, 20 vs. APE0 (upper curve),  $T/T_c = 0.49, 0.57, 0.62, 0.68, 0.76, 0.86$ .

# Triplet free energy



Great improvement with APE10 vs. APE0,  $T/T_c = 0.76$ .  
Comparison to C. Michael, S.J. Perantonis, J. Phys. G 18, 1725  
(1992)

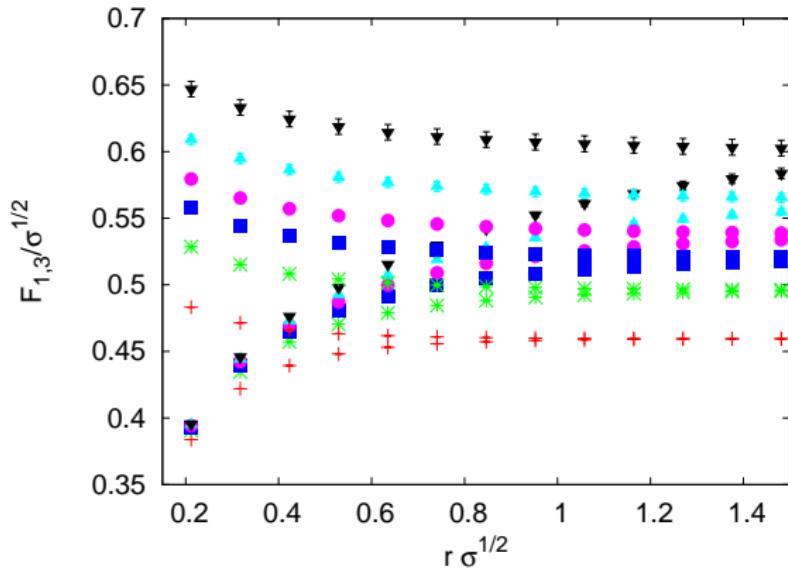
# Triplet free energy (deconfined phase)



Top to bottom  $T/T_c = 0.86, 0.98, 1.14, 1.71, 3.42$ ,  
APE0 vs. APE10.

No dependence on smearing at higher  $T$ .

# Singlet and triplet free energy



Top to bottom  $T/T_c = 0.98, 1.14, 1.37, 1.71, 2.28, 3.42$ ,  
APE20 smearing.

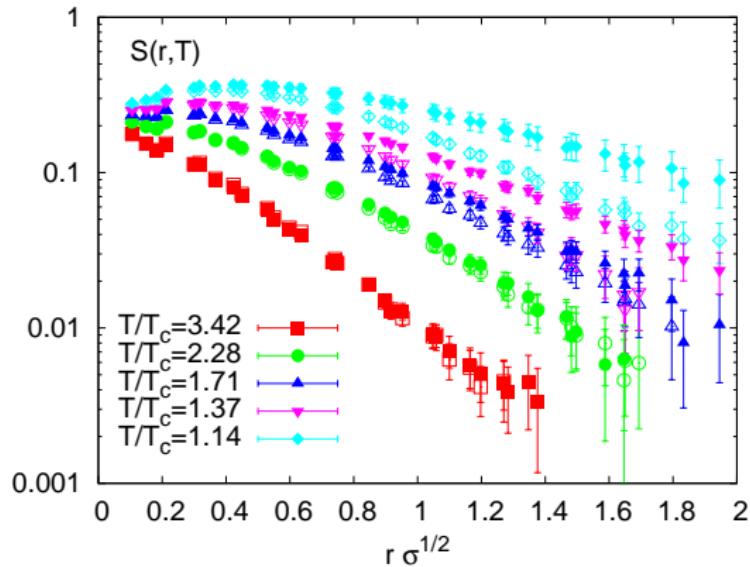
# Screening function

$$S(r, T) = (F_\infty(T) - F_1(r, T)) r = -(r \cdot T) \ln \frac{\langle \text{Tr} L^\dagger(x) L(y) \rangle}{| \langle L \rangle |^2}$$

Its exponential fall-off is governed by the Debye screening mass:

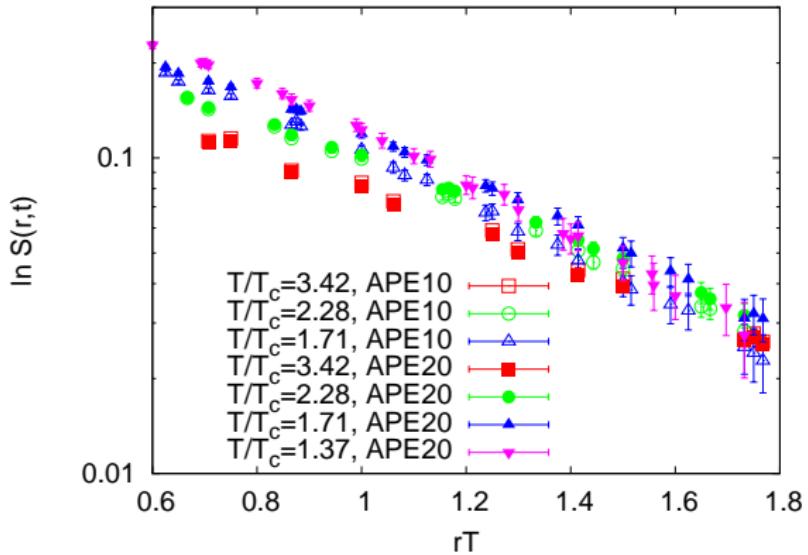
$$\ln S(r, T) \sim -m_D r$$

# Screening function



APE10 vs. APE20 at different temperatures

# Screening function



# Conclusion

- ▶ The singlet, triplet and color averaged static meson correlators calculated using different levels of APE smearing
- ▶ APE smearing procedure allows to remove distance dependence from the matrix elements  $c_1^1$  in the singlet channel thus providing the correct interpretation of the triplet correlator
- ▶ Compared to fixing Coulomb gauge APE smearing procedure offers improvement in assessing triplet free energy contribution of static quark anti-quark pair
- ▶ For higher temperatures dependence on levels of APE smearing vanishes
- ▶ The screening function shows correct exponential fall-off behaviour